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REPORT
CD NO.

50X1-HUM

COUNTRY	USSR	DATE OF INFORMATION	1949
SUBJECT	Theoretical mathematics - Automatic regulation		
HOW PUBLISHED	Bimonthly periodical	DATE DIST.	25 Oct 1949
WHERE PUBLISHED	Moscow	NO. OF PAGES	2
DATE PUBLISHED	Jul/Aug 1949		
LANGUAGE	Russian	SUPPLEMENT TO REPORT NO.	

THIS IS UNEVALUATED INFORMATION

SOURCE Uspekhi Matematicheskikh Nauk, Vol IV, No 4, 1949.

A PROBLEM RELATING TO STABILITY OF DYNAMIC SYSTEMS FOR LARGE INITIAL DEVIATIONS

M. A. Ayzerman

For the importance of this problem to the theory of automatic regulation, see appended bibliography.

We consider a system of linear differential equations

$$dx_1/dt = ax_k + a_{1j}x_j \tag{1}$$

where $a_{1j}x_j$, according to the summation convention, is summed from $j = 1$ to $j = n$;

$$dx_i/dt = a_{ij}x_j \quad (i = 2, 3, \ldots n)$$

where similarly, $a_{ij}x_j$ is summed from $j = 1$ to $j = n$, according to the summation convention.

Let us assume for given constants $a_{p,j}$, and $p, j = 1, 2, \ldots n$ and for any value of a in a certain interval

$$A < a < B \tag{2}$$

that all roots of the characteristic equation of system (1) have real negative parts.

On the basis of a number of examples, the correctness of the following assertion has been assumed:

"For any interval A, B, for which the negative condition of the real parts of the roots of the characteristic equation of system (1) is maintained for a between A and B, and for any unique continuous function f(x) satisfying the conditions:

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(1) for all $x \neq 0$

$$A x < f(x) < Bx$$

and

(2) $f(0) = 0$,

then the system

$$dx_1/dt = f(x_k) + a_{1j}x_j \text{ (summed from } j = 1 \text{ to } j = n)$$
$$dx_1/dt = a_{ij}x_j \text{ (summed from } j = 1 \text{ to } j = n) \qquad (3)$$

whose unique equilibrium state is obviously the origin of the coordinates, $x_1 = x_2 = \ldots = x_n = 0$, will have in the origin of the coordinates a stable equilibrium state and the region of its attraction will encompass all phase spaces of system (3):

$$-\infty < x_i < +\infty \qquad i = 1, 2, \ldots n."$$

We note [1, 2] that, according to Lyapunov's second method, it is possible to select such intervals A', B', so that the assertion is correct, if $f(x)$ satisfies the condition $A'x < f(x) < B'x$ and $f(0) = 0$.

These intervals A', B', however, usually consist of only part of that interval of change of a in which the real parts of the roots of the characteristic equation of system (1) remain negative, i.e., for these selected intervals $A < A' < B' < B$.

The assertion quoted must be either proven or refuted. The more general assertion concerning stability "in the large" of a nonlinear system, obtained from a linear system in the same way as (3) was obtained from (1) except for the introduction of several nonlinear functions in any number of arguments, as shown by G. M. Adel'kon-Vel'skiy, is not correct.

BIBLIOGRAPHY

1. M. A. Ayzerman, "Stability of Systems of Automatic Regulation After Large Initial Deviations," Avtomatika i Telemekhanika, No 6, 1946

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